AK ACADEMICS
CBSE BOARD - 2025-26

CLASS - 12th
PHYSICS

CHAPTER-01
ELECTRIC CHARGE & FIELD

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CHAPTER – 1
ELECTRIC CHARGE AND FIELD

CHARGE AND ITS PROPERTIES

- Electrostatics is the branch of physics that deals with apparently stationary electric charges, that is, with the force exerted by an unchanging electric field upon a charged object.
- Electric charge is the property associated with a body or a particle due to which it is able to produce as well as experience the electric and magnetic effects.

Characteristics of electric charges

- Charge is a fundamental property of matter and never found without matter.
- The excess or deficiency of electrons in responsible for charge on the body. body is
- There are two types of charges namely positive and negative charges.
- The deficiency of electrons in a body is responsible for positive charge.
- The excess of electrons in a body is responsible for negative charge.
- If a body gets positive charge, its mass slightly decreases. If a body is given negative charge, its mass slightly increases.
- Charge is relativistically invariant, i.e. it does not change with motion of charged particle.
  $q_{static} = q_{dynamic}$
- Charge is a scalar quantity S.I. unit of charge is coulomb(C).
  One electrostatic unit of charge
  1 esu = $\frac{1}{3 \times 10^9}$ coulomb.
  One electromagnetic unit of charge (emu) = 10 coulomb
- Charge is a derived physical quantity with dimensions [AT].

Quantization Of Charge

- Quantization of charge means that when we say something has a given charge, mean that how many times the charge of a single electron it has. Because all charges are associated with a whole number of electron that is possible.
- Charge is quantised. The charge on any body is an integral multiple of the minimum charge or electron charge, i.e if q is the charge then $q = \pm ne$ where n is an integer, and e is the charge of electron = 1.6×10^{-19} C
- The minimum charge possible is 1.6×10^{-19} C
- If a body possesses n_1 protons and n_2 electrons, then net charge on it will be $(n_1 - n_2)e$, i.e., $n_1 (e) + n_2(-e) = (n_1 - n_2)e$

Law of Conservation of Charge

Charge is conserved. It can neither be created nor destroyed. It can only be transferred from one object to the other.
The total net charge of an isolated physical system al. ways remains constant,
i.e. $q = q_+ + q_- =$ constant.
In every chemical or nuclear reaction, the total charge be-fore and after the reaction remains constant.
This law is applicable to all types of processes like nuclear, atomic, molecular etc.
Like charges repel each other and unlike charges attract each other.

KEY NOTE

- Charge always resides on the outer surface of a charged conducting body. It accumulates more at sharp points.
- The total charge on a body is algebric sum of the charges located at different points on the body.

ELECTRIFICATION

Methods of Charging a Body

Making a body to acquire property of attracting small objects is called charging(or electrification). A body can be charged by the following ways:

By rubbing:

When a body is rubbed with another body, both of them get charged. One of the two bodies acquires positive charge and the other acquires negative charge. For example, when a glass rod is rubbed with a silk, the glass rod acquires positive charges, and at the same time, the silk acquires negative charges.

By conduction:

When an uncharged conducting body is made in contact with a charged conducting body, charge flows into the uncharged body and the body gets charged. For example, if an uncharged conducting sphere A is made in contact with a charged conducting sphere B, the sphere A will be charged and the sphere A will be charged with the same nature of charge as in the sphere B.

By Induction:

When a charged particle or body is brought near uncharged body without touching, charges developed on the uncharged body. This body. This method of charging a body is called charging by induction.

COULOMB'S LAW AND FORCE DUE TO MULTIPLE CHARGES

- Coulomb's law states that the magnitude of the electrostatic force of attraction or repulsion between two point charges is directly proportional to the product of the magnitudes of charges and inversely proportional to the square of the distance between them.
- F $= \frac{1}{4\pi\epsilon_0\epsilon_r}\frac{q_1q_2}{d^2}$

  ϵ_0 - permittivity of free space or vacuum or air.

  ϵ_r - Relativo permittivity or dielectric constant of medium in which the charges are situated.
- ϵ_0 = 8.857 x 10^{-12} C^2/Nm^2 or farad/metre and $\frac{1}{4\pi\epsilon_0\epsilon_r}$ = 9 x 10^9 Nm^2/C^2
- Suppose the position vector of two charges q_1 and q_2 are $\vec{r_1}$ and $\vec{r_2}$ respectively, then electric force on charge q_1 due to q_2 is,

$$\vec{F_2} = \frac{1}{4\pi\epsilon_0}\frac{q_1q_2}{|\vec{r_1}-\vec{r_2}|^3}(\vec{r_1}-\vec{r_2})$$

  Similarly, electric force on q_2 due to charge q_1 is

$$\vec{F_2} = \frac{1}{4\pi\epsilon_0}\frac{q_1q_2}{|\vec{r_2}-\vec{r_1}|^3}(\vec{r_2}-\vec{r_1})$$

  Here q_1 and q_2 are to be substituted with sign.

  $\vec{r_1} = x_1\hat{\imath} + y_1\hat{\jmath} + z_1\hat{k}$ and $\vec{r_2} = x_2\hat{\imath} + y_2\hat{\jmath} + z_2\hat{k}$ where (x_1, y_1, z_1) and (x_2, y_2, z_2) are the co-ordinates of charges q_1 and q_2 respectively.

Relative permittivity (ϵ_r)

The relative permittivity is the ratio of absolute permittivity of the medium to the absolute permittivity of the free space $\epsilon_r = \in/\epsilon_0$

ϵ_r has no units and dimensional formula is [M^0 L^0 T^0 A^0]

- And also

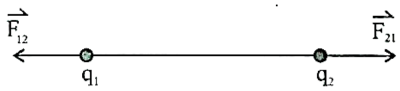
  $\epsilon_r = \frac{\text{Force between two charges in air}}{\text{Force between the same two charges in the medium at same distance}} = \frac{F_{air}}{F_{medium}}$
- For air k = 1

  k > 1 for any dielectric medium;

  k = ∞ for conducting medium like metals

Coulomb's Law In Vector Form

- $\vec{F_{12}} = \frac{1}{4\pi\epsilon_0}\frac{q_1q_2}{{r_{12}}^2}\hat{r}_{12}\ and\ \vec{F_{21}} = -\vec{F_{12}}$



Here $\overrightarrow{F_{12}}$ is force exerted on q_1 due to q_2 and $\overrightarrow{F_{21}}$ force exerted on q_2 due to q_1

- Coulomb's law holds for stationary charges only which are point sized. This law is valid for all types of charge distributions. Coulomb's law is valid at distances greater than 10 m. This law obeys Newton's third law.
  It represents central forces.
  This law is analogous to Newton law of gravitation in mechanics.

KEY NOTE

- The electric force is conservative in nature.
- Coulomb force is central.
- The electric force is an action reaction pair, i.e the two charges exert equal and opposite forces on each other.
- Coulomb force is much stronger than gravitational force.
  (10^{36} $F_g = F_E$)

Forces between multiple charges

To better understand the concept, consider a system of three charges q_1 q_2 and q_3 as shown in Fig (a). The force on one charge, say q_1 due due to two other charges q_2 q_3 can therefore be obtained by performing a vector addition of the forces due to each one of these charges. Thus, if the force on q_1 due to q_2 is denoted by $\overrightarrow{F_{12}}$, $\overrightarrow{F_{13}}$, is given by Eq. (1) even though other charges are present.

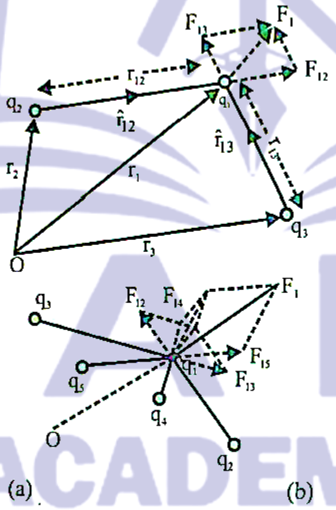


Fig.: A system of (a) three charges (b) multiple charges

Thus, $\overrightarrow{F_{12}} = \frac{1}{4\pi\epsilon_0}\frac{q_1 q_2}{{r_{12}}^2}\hat{r}_{12}$ ....(1)

In the same way, the force on q_1 due to q_3 is given by

$\overrightarrow{F_{13}} = \frac{1}{4\pi\epsilon_0}\frac{q_1 q_2}{{r_{13}}^2}\hat{r}_{13},$ ....(2)

which again is the Coulomb force on q_1 due to q_3 even though other charge q_2 is present.

Thus the total force $\overrightarrow{F_1}$ on q_1 due to the two charges $q_2$and q_3 is given as

$$\overrightarrow{F_1} = \overrightarrow{F_{12}} + \overrightarrow{F_{13}} = \frac{1}{4\pi\epsilon_0}\frac{q_1 q_2}{{r_{12}}^2}\hat{r}_{12} + \frac{1}{4\pi\epsilon_0}\frac{q_1 q_3}{{r_{13}}^2}\hat{r}_{13}$$

The above calculation of force can be generalised to a system of charges more than three, as shown in Fig.(b).

The principle of superposition says that in a system of charges q_1, q_2 ,...,q_n , the force on q_1 due to q_2 is the same as given by Coulomb's law, i.e., it is unaffected by the presence of the other charges q_3, q_4 ,...,q_n . The total force $\overrightarrow{F_1}$ n the charge q_p due to all other charges, is then given by the vector sum of the forces $\overrightarrow{F_{12}}$, $\overrightarrow{F_{13}}$,..., $\overrightarrow{F_{In}}$

i.e.

$\overrightarrow{F_1} = \overrightarrow{F_{12}} + \overrightarrow{F_{13}} + ... + \overrightarrow{F_{In}} = \frac{1}{4\pi\epsilon_0}\left[\frac{q_1 q_2}{{r_{12}}^2}\hat{r}_{12} + \frac{q_1 q_3}{{r_{13}}^2}\hat{r}_{13} \;.....+ \frac{q_1 q_n}{{r_{1n}}^2}\hat{r}_{1n}\right]$

$= \frac{q_1}{4\pi\epsilon_0} \sum_{i=2}^{n} \frac{q_i}{r^2{}_{1i}} \hat{r}_{1i}$ ...(4)

The vector sum is obtained as usual by the parallelogram law of addition of vectors. All of electrostatics is basically a consequence of Coulomb's law and the superposition principle.

- If the force between two charges in different media is the same for different separations, then

  $$F = \frac{1}{k} \frac{1}{4\pi \in_0} \frac{q_1 q_2}{d^2} = \text{constant.}$$

  $$kd^2 = \text{constant or } k_1 d_1^2 = k_2 d_2^2$$

- If two charged spheres of radii r_1 and r_2 respectively are kept in contact for some time, then charge exchanges takes place between them until their electric potentials are equal. Now the new charges on two spheres are

  $$q_1 = \left(\frac{r_1}{r_1 + r_2}\right) q \ \& \ q_2 = \left(\frac{r_2}{r_1 + r_2}\right) q$$

  where 'q' is the total charge on the two spheres
- Two point sized identical spheres carrying charges q_1 and q_2 on them are seperated by a certain distance. The mutual force between them is F. These two are brought in contact and kept at the same separation. Now, the force between them is F^1 Then

  $$\frac{F^1}{F} = \frac{(q_1 + q_2)^2}{4q_1 q_2}$$

- Two identical charged spheres are suspended by strings of equal lengths in gravitational field. The strings make an angle of 'θ' with each other due to repulsion. When same system is kept immersed in a liquid of density 'd_l', even then the angle between them remains same. Then the dielectric constant of the liquid,

  $$K = \frac{d_b}{d_b - d_\ell}$$

  Where d_b - density of the body.

  Note: If gravitational field is absent, then the angle between two strings is 180°.

- **Test charge**

  That small positive charge, which does not affect the other charges present and by the help of which we determine the effect of other charges, is defined as test charge.

- **Linear charge density (λ)** is defined as the charge per unit length.

  $$\lambda = \frac{dq}{dl}$$

  where dq is the charge on an infinitesimal length dl.

  Unit of λ is Coulomb/meter (C/m)

  Examples: Charged straight wire, circular charged ring

- **Surface charge density (σ)** is defined as the charge per unit area.

  $$\sigma = \frac{dq}{ds}$$

  where dq is the charge on an infinitesimal surface area ds. Unit of (σ) is coulomb/meter^2 (C/m^2).

  Examples: Plane sheet of charge, conducting sphere.

- **Volume charge density (p)** is defined as charge per unit volume.

  $$\rho = \frac{dq}{dv}$$

  where dq is the charge on an infinitesimal volume element dv. Unit of (p) is coulomb/meter^3 (C/m^3)

  Examples: Charge on a dielectric sphere etc.
- Charge given to a conductor always resides on its outer surface.
- If surface is uniform then the charge distributes uniformly on the surface.

KEY NOTE

- In conductors having nonspherical surfaces, the surface charge density (σ) will be larger when the radius of curvature is small.
- The working of lightening conductor is based on leakage of charge through sharp point due to high surface charge density.

ELECTRIC FIELD

- The space around electric charge upto which its influence is felt is known as electric field.
- Electric field is a conservative field.

Lines of Force

- Line of force is an imaginary path along which a unit +ve test charge would tend to move in an electric field.

Characteristics of line of force

- Lines of force start from +ve charge and end at-ve charge.
- Lines of force in the case of isolated ve charge are radially outwards and in the case of isolated -ve charge are radially inwards.
- The tangent at any point to the curve gives the direction of electric field at that point.
- Lines of force do not intersect.
- Lines of force won't pass through a conductor.
- The electric lines of force are perpendicular to equipotential surface.
- The force experienced by a unit positive test charge placed at a point in the electric field gives the intensity of electric field at that point both in magnitude and direction.

Difference Between Electric And Magnetic Lines Of Force

- The difference between electric lines of force and magnetic lines of field is magnetic field lines are closed curves and electric lines of force have a beginning and ending.
- Electric lines of force do not exist inside a conductor, but magnetic lines of force may exist inside a magnetic material
- Total electric lines of force linked with a closed surface may or may not be zero, but total magnetic field lines linked with a closed surface is always zero (as monopoles do not exist).
- Intensity of electric field is a vector quantity. Its direction is always away from the positive charge and towards the negative charge.
- Intensity of electric field at a point which is at a distance 'd' from the point charge 'Q' in air is

  $$E_0 = \frac{1}{4\pi \epsilon_0} \frac{Q}{d^2}$$

  and in a medium

  $$E = \frac{1}{k} \frac{1}{4\pi \epsilon_0} \frac{Q}{d^2} = E_0 / k$$

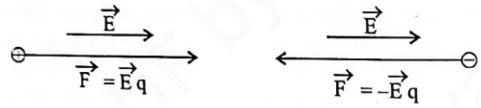
- S.I. unit is newton/coulomb (NC^{-1}) volt/metre (Vm^{-1}). Dimensional formula $MLT^{-3}A^{-1}$
- In vector form

  $$\vec{E} = \frac{1}{4\pi\varepsilon_0} \frac{q}{r^3} \vec{r}$$

- A charge in an electric experiences a force whether it is at rest or moving.
- The electric force is independent of mass and velocity of the charged particle. It depends upon the charge.
- If instead of a single charge, field is produced by no. of charges, by the principle of super position resultant electric field intensity

  $$\vec{E} = \vec{E}_1 + \vec{E}_2 + \vec{E}_3 + ...$$

- If q_0 is positive charge then the force acting on it is in the direction of the field.
- If q_0 is negative then the direction of this force is opposite of the field direction.



Motion of a charged particle in a uniform electric field

A charged body of mass 'm' and charge "q" is initially at rest in a uniform electric field of intensity E. The force acting on it F = Eq

- Here the direction of F is in the direction of field if "q' is +ve and opposite to the field if 'q' is -ve.
- The body travels in a straight line path with uniform acceleration, a = F/m = (Eq)/m, initial velocity u = -0.

  At an instant of time t,

  Its final velocity,

  $$v = u + at = \left(\frac{Eq}{m}\right)t$$

  Displacement

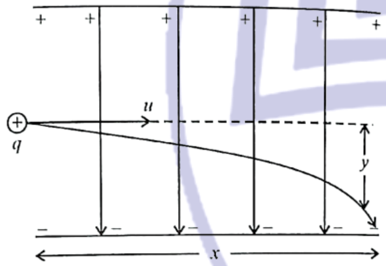
  $$s = ut + \frac{1}{2}at^2 = \frac{1}{2}\left(\frac{Eq}{m}\right)t^2$$

  Momentum, P = mv = (Eq)t

  Kinetic energy,

  $$K.E = \frac{1}{2}mv^2 = \frac{1}{2}\left(\frac{E^2q^2}{m}\right)t^2$$

- When a charged particle enters perpendicularly into a uniform electric field of intensity E with a velocity v then it describes parabolic path as shown in figure.



Along the horizontal direction, there is no acceleration and hence x = ut

Along the vertical direction, acceleration

a = F/m = (Eq)/m (here gravitational force is not considered)

Hence vertical displacement,

$$y = \frac{1}{2}\left(\frac{Eq}{m}\right)t^2$$

$$y = \frac{1}{2}\left(\frac{qE}{m}\right)\left(\frac{x}{u}\right)^2 = \left(\frac{qE}{2mu^2}\right)x^2$$

- At any instant of time t, horizontal component of velocity, $v_x = u$
- vertical componet of velocity

$$v_y = at = \left(\frac{Eq}{m}\right)t$$

$$\therefore v = |\vec{v}| = \sqrt{v_x^2 + v_y^2} = \sqrt{u^2 + \frac{E^2q^2t^2}{m^2}}$$

Null Points (Or) Neutral Points

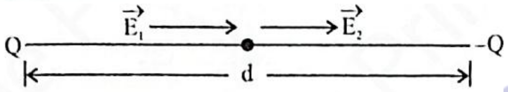
Two charges q_1 and q_2 are seperated by a distance 'd'. Then the point of zero intensity (null point) lies at a distance of

$$x = \frac{d}{\sqrt{\frac{q_2}{q_1}} \pm 1} \text{ from } q_1$$

from q_1 +ve sign for like charges-ve sign for unlike charges

- **Some important considerations**
  - Two charges +Q and Q are separated by a distance 'd'. The intensity of electric field at the mid-point of the line joining the charges is

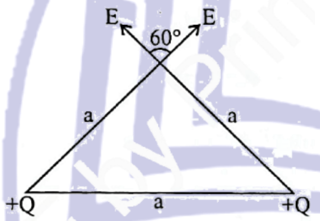
    $$\vec{E} = \vec{E}_1 + \vec{E}_2$$

    $$E = \frac{1}{4\pi \epsilon_0} \frac{8Q}{d^2}$$

  - Two charges +Q each are separated by a distance 'd'. The intensity of electric field at the mid point of the line joining the charges is zero.
  - Two charges +Q each are placed at the two vertices of an equilateral triangle of side a. The intensity of electric field at the third vertex is

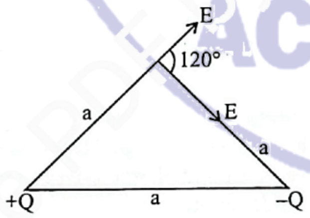
    $$E' = 2E\cos\frac{\theta}{2} = \sqrt{3}E \quad (\because \theta = 60°)$$

    $$E' = \sqrt{3}\frac{1}{4\pi \epsilon_0}\frac{Q}{a^2}.$$

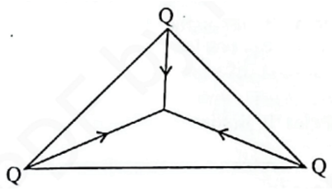
  - Two charges +Q, Q are placed at the two vertices of an equilateral triangle of side 'a', then the intensity of electric field at the third vertex is

    $$E' = 2E\cos\frac{\theta}{2} = E \; (\theta = 120°)$$

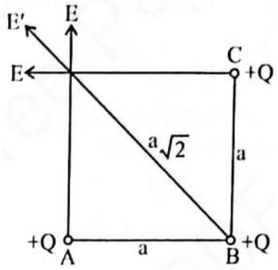
    

    $$E' = \frac{1}{4\pi \epsilon_0}\frac{Q}{a^2}.$$

  - If three charges +Q each are placed at the three vertices of an equilateral triangle of side 'a' then the intensity of electric field at the centroid is zero.

  - If three charges +Q each are placed at the three corners of a square of side 'a' as shown in figure.



Intensity of electric field at the fourth corner = $\sqrt{2}E + E'$

Where $E = \frac{1}{4\pi \epsilon_0}\frac{Q}{a^2}$ and $E' = \frac{1}{4\pi \epsilon_0}\frac{Q}{2a^2} = \frac{E}{2}$

Hence the intensity of electric field at the fourth corner

$= E\left(\sqrt{2} + \frac{1}{2}\right)$

- When a charged particle of mass m and charge Q remains suspended in an electric field then mg = EQ
- When a charged particle of mass m and charge Q remains suspended in an electric field, the number of fundamental charges on the charged particle,
  mg = EQ = E(ne)
  n = (mg)/(Ee)

KEY NOTE

A charged particle of charge plus/minus Q is projected with an initial velocity u in a vertically upward electric field E making an angle 0 to the horizontal. Then

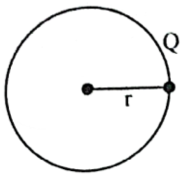
Time of flight $= \dfrac{2u\sin\theta}{g \mp \dfrac{EQ}{m}}$

Maximum height $= \dfrac{u^2\sin^2\theta}{2\left(g \mp \dfrac{EQ}{m}\right)}$

Range $= \dfrac{u^2\sin 2\theta}{g \mp \dfrac{EQ}{m}}$

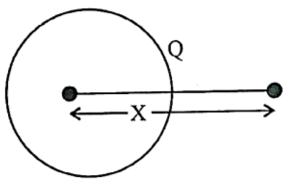
- Intensity of electric field inside a charged hollow conducting sphere is zero.
- A hollow sphere of radius r is given a charge Q. Intensity of electric field at any point inside it is zero. Intensity of electric field on the surface of the sphere is

$\frac{1}{4\pi \epsilon_0}\frac{Q}{r^2}$



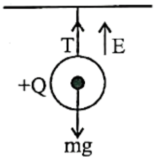
Intensity of electric field at any point outside the sphere is (at a distance 'x' from the centre)

$\frac{1}{4\pi \epsilon_0}\frac{Q}{x^2}$



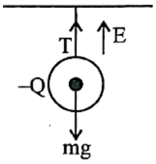
- The bob of a simple pendulum is given a +ve charge and it is made to oscillate in a vertically upward electric field, then the time period of oscillation is

$$2\pi\sqrt{\frac{l}{g-\frac{EQ}{m}}}$$



- In the above case, if the bob is given a -ve charge then the time period is given by

$$2\pi\sqrt{\frac{l}{g+\frac{EQ}{m}}}$$



- A sphere is given a charge of Q' and is suspended in a horizontal electric field. The angle made by the string with the vertical is,

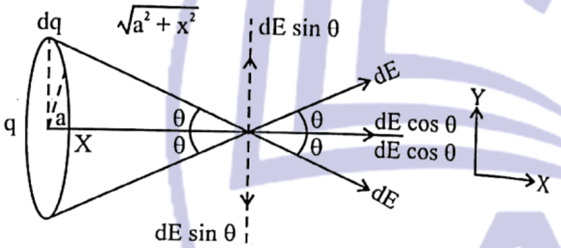
  $$\theta = \tan^{-1}\left(\frac{EQ}{mg}\right)$$

- The tension in the string is

  $$\sqrt{(EQ)^2+(mg)^2}$$

- A bob carrying a +ve charge is suspended by a silk thread in a vertically upward electric field, then the tension in the string is, T = mg - EQ
- If the bob carries -ve charge, tension in the string is mg + EQ

Electric Field at the Axis of a Circular Uniformly Charged Ring



Intensity of electric field at a point P that lies on the axis of the ring at a distance x from its centre is

$$E=\frac{1}{4\pi\varepsilon_0}\frac{qx}{\left(x^2+R^2\right)^{3/2}} \qquad \text{where } \left\{\cos\theta=\frac{x}{\sqrt{a^2+x^2}}\right\}$$

Where R is the radius of the ring. From the above expression E = 0 at the centre of the ring.

E will be maximum when $\frac{dE}{dx}=0$

Differentating E w.r.t x and putting it equal to zero we get

$$x=\frac{R}{\sqrt{2}} \text{ and } E_{\max}=\frac{2}{3\sqrt{3}}\left(\frac{1}{4\pi\varepsilon_0}\frac{q}{R^2}\right)$$

Electric Field Due To A Charged Spherical Conductor (Spherical Shell)

'q' amount of charge be uniformly distributed over a spherical shell of radius 'R'

σ = Surface charge density,

$$\sigma=\frac{q}{4\pi R^2}$$

When point 'P' lies outside the shell:

$$E=\frac{1}{4\pi\epsilon_0}\times\frac{q}{r^2}$$

This is the same expression as obtained for electric field at a point due to a point charge. Hence a charged spherical shell behave as a point charge concentrated at the centre of it, for outside point.

$$E = \frac{1}{4\pi \epsilon_0} \frac{\sigma . 4\pi R^2}{r^2} \quad \because \sigma = \frac{q}{4\pi R^2}$$

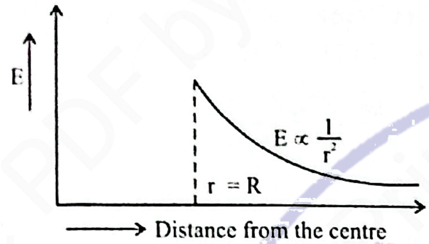
$$E = \frac{\sigma . R^2}{\epsilon_0 \, r^2}$$

When point 'P' lies on the shell:

$$E = \frac{\sigma}{\epsilon_0}$$

When Point 'P' lies inside the shell:

E = 0



Note: The field inside the cavity of a metallic body is always zero. This is known as electrostatic shielding.

- **Electric Field Due To A Uniformly Charged Non Conducting Sphere**

Electric field intensity due to a uniformly charged non-conducting sphere of charge Q, of radius R, at a distance r from the centre of the sphere.

Q is the amount of charge, uniformly distributed over a solid sphere of radius R.

ρ = Volume charge density

$$\rho = \frac{Q}{\frac{4}{3}\pi R^3}$$

When point 'P' lies inside sphere:

$$E = \frac{1}{4\pi\varepsilon_0} \frac{Qr}{R^3} \quad \text{for } r < R$$

$$E = \frac{\rho . r}{3 \epsilon_0}$$

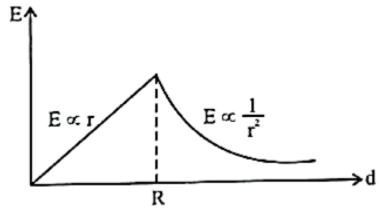
When point 'P' lies on the sphere:

$$E = \frac{1}{4\pi\varepsilon_0} \frac{Q}{R^2} \;;\; E = \frac{\rho . R}{3 \epsilon_0}$$

When point 'P' lies outside the sphere:

$$E = \frac{1}{4\pi \epsilon_0} \frac{Q}{r^2}$$

$$E = \frac{\rho . R^3}{3 \epsilon_0 \, r^2}$$



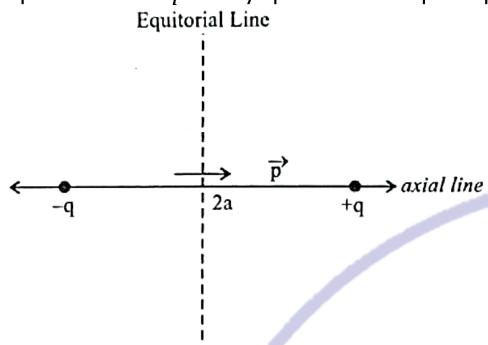
- **ELECTRIC DIPOLE**

An electric dipole is a pair of equal and opposite point charge qand -q separated by a small distance 2a. The line connecting the two charges defines a direction in space. The direction from -q to g is said to be axis of the dipole. The total charge of the electric dipole is zero.

Dipole moment (p)

It is defined as the product of magnitude of either charge and the distance of separation between the two charges. $\vec{p} = q(2\,\vec{a}\,)$ 2a is the distance between the two charges.)

Dipole moment $\vec{p}$ always points from -q to +q.



Electric Dipole

Derivation of the field of an electric dipole

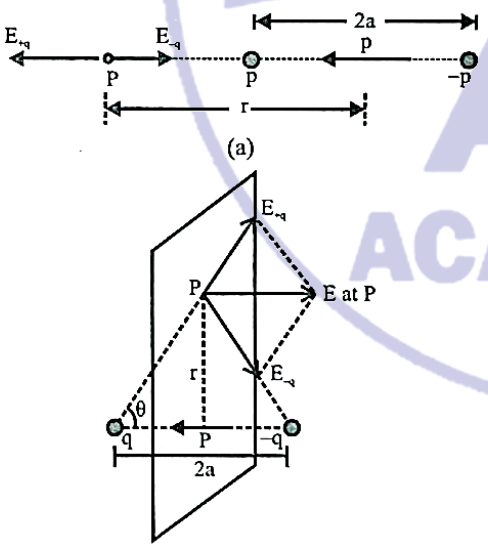
(i) For points on the axis

Let the point P be at distance r from the centre of the dipole on the side of the charge q, as shown in Fig.(a). Then

$$E_{-q} = -\frac{q}{4\pi\varepsilon_0 (r+a)^2}\hat{p} \quad ...(1)$$

where p is the unit vector along the dipole axis (from -q to q). Also

$$E_{+q} = +\frac{q}{4\pi\varepsilon_0 (r-a)^2}\hat{p} \quad ...(2)$$



(a)

(b)

Fig.: Electric field of a dipole at (a) a point on the axis, (b) a point on the equatorial plane of the dipole $\vec{p}$ is the dipole moment vector of magnitude p = q x 2a and directed from -q to q

The total field at P is

$$E = E_{+q} + E_{-q} = \frac{q}{4\pi\varepsilon_0}\left[\frac{1}{(r-a)^2} - \frac{1}{(r+a)^2}\right]\hat{p}$$

$$= \frac{q}{4\pi\varepsilon_0} \frac{4ar}{\left(r^2 - a^2\right)^2} \hat{p} \qquad ...(3)$$

For r >> a

$$E = \frac{4qa}{4\pi\varepsilon_0 r^3} \hat{p} \quad (r >> a) \qquad ...(4)$$

(ii) For points on the equatorial plane

The magnitudes of the electric fields due to the two charges +q and -q are given by

$$E_{+q} = \frac{q}{4\pi\varepsilon_0} \frac{1}{r^2 + a^2} \qquad ...(5)$$

$$E_{-q} = \frac{q}{4\pi\varepsilon_0} \frac{1}{r^2 + a^2} \qquad ...(6)$$

and are equal.

The directions of E_{+q} and E_{-q} are as shown in Fig.(b). Clearly, the components normal to the dipole axis cancel away. The components along the dipole axis add up. electric field is opposite to $\hat{p}$. We have

$$E = -\left(E_{+q} + E_{-q}\right)\cos\theta \hat{p}$$

$$= -\frac{2qa}{4\pi\varepsilon_0 \left(r^2 + a^2\right)^{3/2}} \hat{p} \qquad ...(7)$$

At large distances (r >> a), this reduces to

$$E = -\frac{2qa}{4\pi\varepsilon_0 r^3} \hat{p} \quad (r >> a) \qquad ...(8)$$

From Eqs. (4) and (8), it is clear that the dipole field at large distances does not involve q and a separately; it depends on the product qa. This suggests the definition of dipole moment. The dipole moment vector p of an electric dipole is de-fined by

$$\vec{p} = q \; x \; 2a\hat{p} \qquad ...(9)$$

that is, it is a vector whose magnitude is charge q times the separation 2a (between the pair of charges q, -q) and the direction is along the line from -q to q. In terms of p, the electric field of a dipole at large distances takes simple forms:

At a point on the dipole axis

$$E = +\frac{2\vec{p}}{4\pi\varepsilon_0 r^3} \quad (r >> a) \qquad ...(10)$$

At a point on the equatorial plane

$$E = -\frac{\vec{p}}{4\pi\varepsilon_0 r^3} \quad (r >> a)$$

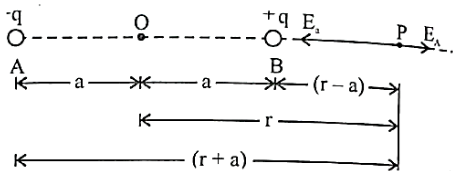
- **Electric Field Due To A Dipole At A Point Lying On The Axial Line (End On Position)**

$$E_{axial} = \frac{1}{4\pi\varepsilon_0} \frac{2pr}{\left(r^2 - a^2\right)^2}$$

(from negative to positive charge)

In case of a short dipole (r>>a).

$$E_{axial} = \frac{1}{4\pi\varepsilon_0} \frac{2p}{r^3}$$



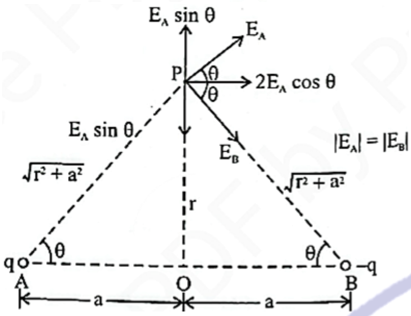
- **Electric field due to a dipole at a point lying on the equatorial line (Broad side on position)**

$$E_{equatorial} = \frac{1}{4\pi\varepsilon_0}\frac{p}{\left(r^2+a^2\right)^{3/2}}$$

{from positive to negative charge}

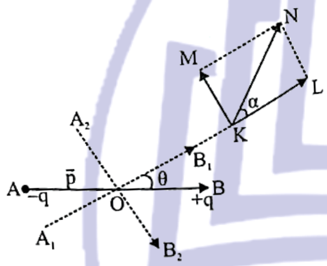
In case of short dipole (r>>a),

$$E_{equatorial} = \frac{1}{4\pi\varepsilon_0}\frac{p}{r^3}$$



➤ Electric field due to a short dipole at any point K(r, θ)

Figure AB represents a short electric dipole of moment $\vec{p}$ along $\overrightarrow{AB}$ O is the centre of dipole. We have to calculate electric field $\vec{E}$ intensity at any point K, where



OK = r, $\angle BOK = \theta$

The dipole moment $\vec{p}$ can be resolved into two rectangular components:

(p cosθ) along A_1B_1 and (p sinθ) along $A_2B_2 \perp A_1B_1$. Field intensity at K on the axial line of A_1B_1

$$\left|\vec{E}_1\right| = \frac{2p\cos\theta}{4\pi\epsilon_0 r^3}$$

Let it be represented by $\overrightarrow{KL}$ along OK. Field intensity at K on equatorial line of A_2B_2

$$\left|\vec{E}_2\right| = \frac{p\sin\theta}{4\pi\epsilon_0 r^3}$$

Let it be represented by $\overrightarrow{KM}$ || B_2A_2. Complete the rectangle KLNM. According to || gm law, $\overrightarrow{KN}$ represents resultant intensity ($\vec{E}$) at K due to the short dipole.

$$\text{As} \quad KN = \sqrt{KL^2 + KM^2}$$

$$\therefore |\vec{E}| = \sqrt{E_1^2 + E_2^2} = \sqrt{\left(\frac{2p\cos\theta}{4\pi\epsilon_0 r^3}\right)^2 + \left(\frac{p\sin\theta}{4\pi\epsilon_0 r^3}\right)^2}$$

$$= \frac{p}{4\pi\epsilon_0 r^3}\sqrt{4\cos^2\theta + \sin^2\theta}$$

$$|\vec{E}| = \frac{p}{4\pi\epsilon_0 r^3}\sqrt{3\cos^2\theta + \left(\cos^2\theta + \sin^2\theta\right)}$$

$$= \frac{p}{4\pi\epsilon_0 r^3}\sqrt{3\cos^2\theta + 1}$$

$$\text{i.e.,} \quad |\vec{E}| = \frac{p\sqrt{3\cos^2\theta + 1}}{4\pi\epsilon_0 r^3}$$

Let $\angle LKN = \alpha$

In ΔKLN, $\tan\alpha = \frac{LN}{KL} = \frac{KM}{KL} = \frac{p\sin\theta}{4\pi\epsilon_0 r^3}\cdot\frac{4\pi\epsilon_0 r^3}{2p\cos\theta}$

or $\tan\alpha = \frac{1}{2}\tan\theta$

$\therefore$ α can be calculated

- **Particular Cases:**

1. When the point K lies on axial line of dipole.

$$\theta = 0°,\ \cos\theta = \cos 0° = 1$$

$$\therefore |\vec{E}| = \frac{p}{4\pi\epsilon_0 r^3}\sqrt{3\cos^2 0° + 1} = \frac{2p}{4\pi\epsilon_0 r^3} \text{ and}$$

$$\tan\alpha = \frac{1}{2}\tan 0° = 0,\ \therefore \alpha = 0°$$

i.e., resultant intensity is along the axial line.

2. When the point K lies on equatorial line of dipole.

$$\theta = 90°,\ \cos\theta = \cos 90° = 0$$

$$\therefore E = \frac{p}{4\pi\epsilon_0 r^3}\sqrt{3\cos^2 90° + 1} = \frac{p}{4\pi\epsilon_0 r^3} \text{ and}$$

$$\tan\alpha = \frac{1}{2}\tan 90° = \infty,\ \theta = 90°$$

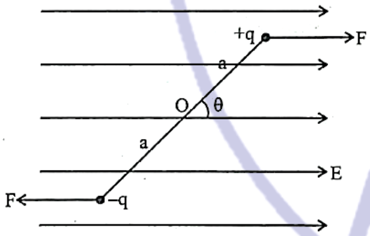
i.e., direction of resultant field intensity is perpendicular to the equatorial line (and hence antiparallel to axial line of dipole). We have already proved these results.

- **Torque on a dipole placed in a uniform electric field :**

The torque due to the force on the positive charge about a point O is given by Fa sin θ The torque on the negative charge about O is also Fa sinθ

$\tau = 2$ Fa sin q

$\Rightarrow \tau = 2aq E\sin\theta \Rightarrow \tau = pE\sin\theta$



$\vec{\tau} = \vec{p}\times\vec{E}$

- **KEY NOTE**

Let electric force between two dipoles be F, then

$F = \frac{1}{4\pi\epsilon_0}\frac{6p_1p_2}{r^4}$ when dipoles are placed coaxially to each other.

$F = \frac{1}{4\pi\epsilon_0}\frac{6p_1p_2}{r^4}$ when dipoles are placed perpendicular to each other.

Work done in rotating a dipole in a uniform electric field

When an electric dipole is placed in a uniform electric field E, a torque, $\vec{\tau} = \vec{p}\ x\ \vec{E}$ acts on it. If we rotate the dipole through. a small angle de as in the Fig. (b) the work done by the torque is

$$dW = \tau d\theta$$

$$dW = -pEsin\theta d\theta$$

The work is negative as the rotation dθ is opposite to the torque.

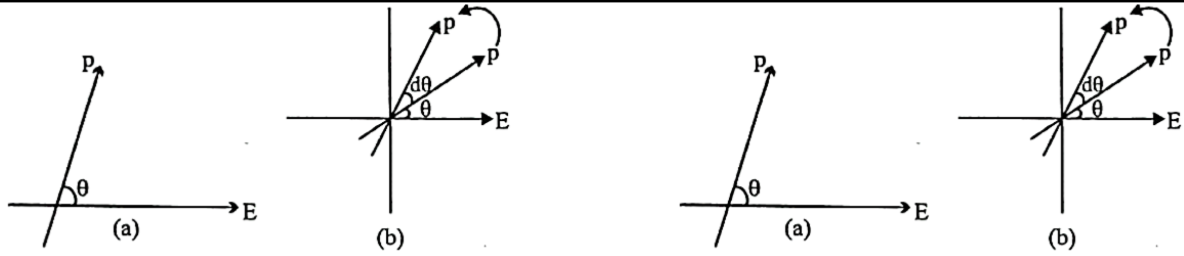


Fig.: Dipole at different angles with electric field

Total work done by external forces in rotating a dipole from,

$\theta = \theta_1$ to $\theta = \theta_2$ will be given by

$$W = \int_{\theta_1}^{\theta_2} pE\sin\theta \, d\theta$$

$$W_{\text{external forces}} = pE\,(\cos\theta_1 - \cos\theta_2)$$

and work done by electric forces,

$$W_{\text{electric force}} = -W_{\text{external force}} = pE(\cos\theta_2 - \cos\theta_1)$$

If Taking $\theta_1 = \theta$ and $\theta_2 = 90°$

We have,

$$W_{\text{electric dipole}} = p.\,E\,(\cos 90° - \cos\theta) = -pE\cos\theta$$

$$= -\vec{p}.\vec{E}$$

GAUSS'S LAW

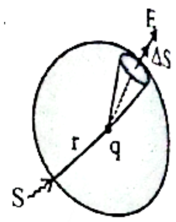
According to Gauss's law, "the net electric flux through any closed surface is equal to the net charge enclosed by it divided by ϵ_0" Mathematically, it can be written as

$$\phi_E = \oint \vec{E} \cdot d\vec{S} = \frac{q}{\varepsilon_0}$$

Derivation of Gauss's Law

Let a point charge +q be placed at centre O of a sphere S. Then S is a Gaussian surface. Electric field at any point on S is given by

$$E = \frac{1}{4\pi\varepsilon_0} \cdot \frac{q}{r^2}$$



The electric field and area element points radially out-wards. So

$\theta = 0°$.

Flux through area $\vec{dS}$ is

$$d\phi = \vec{E}.\vec{dS} = EdS\cos 0° = EdS$$

Total flux through surface S is

$$\phi = \oint_S d\phi = \oint_S EdS = E\oint_S dS = E \times \text{Area of Sphere}$$

$$\frac{1}{4\pi\varepsilon_0} \cdot \frac{q}{r^2} \cdot 4\pi r^2 \quad \text{or,} \quad \phi = \frac{q}{\varepsilon_0} \text{ which proves Gauss's theorem.}$$

Continuous charge distribution

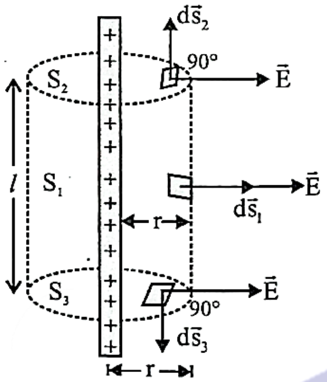
Linear charge distribution: q = λι Where λ = linear charge density.

Surface charge distribution: q = σA where = surface charge density.

Volume charge distribution: q = pA where p = volume charge density.

Applications of Gauss's Law

(i) Electric field due to an infinitely long charged wire.



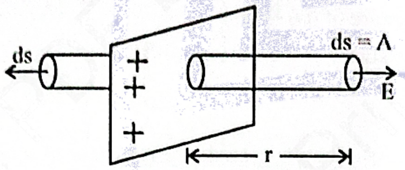
$\phi_E = \oint \vec{E}.\vec{ds} = \int E.dS_1 \cos 0^\circ = E \times 2\pi rl$

(all other becomes zero as $\theta = 90°$)

Using Gauss law, also $\phi_E = \frac{q}{\varepsilon_0}$

$E.2\pi rl = \frac{\lambda l}{\varepsilon_0} \Rightarrow E = \frac{\lambda}{2\pi\varepsilon_0 r}$

(ii) Electric field intensity due to a uniformly charged infinite plane sheet.



$\phi_E = \oint \vec{E}.\vec{ds} = 2EA$ (due to both circular surface)

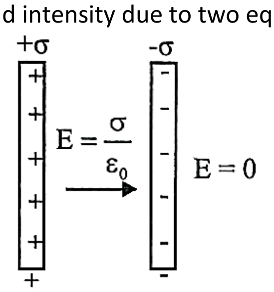
$q = \sigma A$

Using Gauss's law, $\phi_E = \frac{q}{\varepsilon_0}$

$2EA = \frac{\sigma A}{\varepsilon_0} \Rightarrow \boxed{E = \frac{\sigma}{2\varepsilon_0}}$

Note: Electric field is independent or r.

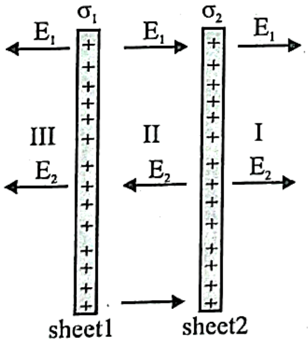
(iii) Electric field intensity due to two equally and oppositely charged parallel plane sheets of charge.



$E = \frac{\sigma}{\varepsilon_0}$ (between two plates)

$E = 0$ (outside the plates)

(iv) Electric field intensity due to two positively charged parallel plane sheets of charge.



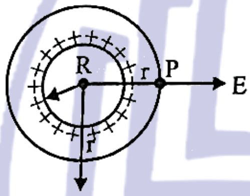
Consider, $\sigma_1 > \sigma_2 > 0$;

In region I: $E_{net} = E_1 + E_2 = \frac{(\sigma_1 + \sigma_2)}{2\varepsilon_0}$ (towards right)

In region II: $E_{net} = E_1 - E_2 = \frac{(\sigma_1 - \sigma_2)}{2\varepsilon_0}$ (towards right)

In region III: $E_{net} = E_1 + E_2 = \frac{(\sigma_1 + \sigma_2)}{2\varepsilon_0}$ (towards left)

(v) Electric field due to uniformly charged thin spherical shell.



Inside the sphere $(r < R)$

By Gauss's law, $\oint \vec{E}.\vec{ds} = \frac{q}{\varepsilon_0}$

$E = 0$ (as $q = 0$ inside)

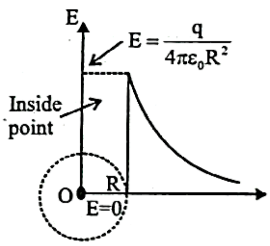
Outside the sphere $(r > R)$

By Gauss's law, $\oint \vec{E}.\vec{ds} = \frac{q}{\varepsilon_0} = \frac{\sigma A}{\varepsilon_0}$

$E \times 4\pi r^2 = \frac{\sigma \times 4\pi R^2}{\varepsilon_0}; E = \frac{\sigma R^2}{\varepsilon_0 r^2}$

$E = \frac{1}{4\pi\varepsilon_0} \cdot \frac{q}{r^2}$

$E \propto \frac{1}{r^2}$



On the surface $(r = R)$

$E = \frac{\sigma R^2}{\varepsilon_0 r^2}$

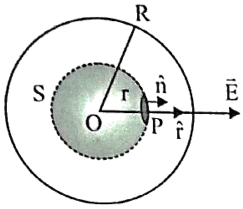
For $r = R$,

$E = \frac{\sigma}{\varepsilon_0} = \frac{q}{4\pi\varepsilon_0 R^2}$

- **Electric Field Intensity Due to a Non-conducting Charged Solid Sphere**

Suppose a non conducting solid sphere of radius R and centre O has uniform volume density of charge p.

We have to calculate electric field intensity $\vec{E}$ at any point P, where OP = r With O as centre and r as radius, imagine a sphere S, which acts as a Gaussian surface, Fig.. At every point of S, magnitude of $\vec{E}$ is same, directed radially outwards.



If q' is the charge enclosed by the sphere S, then according to Gauss's law

$$\oint_S \vec{E}.\vec{ds} = \oint_S \vec{E}.\hat{n}\, ds = E\oint_S ds = \frac{q'}{\in}$$

where e is electrical permittivity of the material of the insulating sphere.

$$\therefore \quad E\left(4\pi r^2\right) = \frac{q'}{\in} \quad \text{or} \quad E = \frac{q'}{4\pi \in r^2} \qquad ...(1)$$

Now, charge inside S. i.e., q ,= volume of Sx volume density of charge

$$q' = \frac{4}{3}\pi r^3 \times \rho$$

$$\text{From (2),}\quad E = \frac{4}{3}\frac{\pi r^3 \rho}{4\pi \in r^2} = \frac{r\rho}{3\in}, \quad \text{i.e.,}\quad E = \frac{r\rho}{3\in} \qquad ...(2)$$

Clearly, $\quad E \propto r$

i.e., electric intensity at any point inside a non-conducting charged solid sphere varies directly from the centre of the sphere. as the distance of the point

At the centre of the sphere, r = 0 .

$$\therefore \qquad E = 0$$

Also, $\quad E = \frac{R\rho}{3\in} \Rightarrow$ maximum at surface of sphere

We have already proved that outside the sphere, $E \propto 1 / r^2$

All these results are plotted in fig., which represents the variation of electric field intensity E with distance (r) from the centre of a non conducing uniformly charged solid sphere.